

C. L. Chao  
 TRW Systems Group  
 One Space Park  
 Redondo Beach, California 90278

Abstract

This paper presents an N-way branch line coupler which is capable of splitting an input signal into N-way equal or unequal amplitude output signals. It consists of N parallel and 2(N-1) shunt quarter wavelength transmission lines and gives a reasonable isolation among output ports at the design center frequency.

The purpose of this paper is to describe and analyze an N-Way branch line directional coupler which is capable of splitting an input signal into any number of equal or unequal amplitude output signals. In the past, branch line directional hybrids have been studied with symmetrical transmission line structure. The symmetrical branch line hybrid with equal power split was analyzed previously using the method of even and odd mode excitation. This method has been discussed in detail by Reed and Wheeler<sup>1</sup> for the case of four-port symmetrical structure. Later Young<sup>2</sup> presented an analysis in designing broad-band symmetrical branch line hybrids allowing unequal as well as equal power split. Design data for multisection branch line hybrid having maximally flat or almost exact chebyshev characteristics were given by Levy and Lind<sup>3</sup>. Recently, Kuroda, et al<sup>4</sup> proposed a multi-port matrix type coupler, which provides any number of input and output ports, and presented numerically analyzed data for 3- and 4-way power division.

In this paper an N-way symmetrical branch line coupler is described. Two types of the N-way coupler are illustrated in Fig. 1 and 2. Fig. 1 (a) shows the line structure which consists of N parallel and 2(N-1) shunt transmission lines. All lines are a quarter wave in length at the design center frequency. The characteristic impedance of the parallel and shunt lines is indicated in Fig. 1 (a). Power entering the port 1 splits equally at the ports 1' through N' when all ports are terminated with 1 ohm resistors. Since the coupler structure is symmetrical with respect to the center line 1-1', it is obvious that all ports 2 through N have equal potentials. Likewise, ports 2' through N' have equal potentials. Therefore, the 2N-port network can be reduced to an equivalent four-port network as shown in Fig. 1 (b). Fig. 2 (a) shows an alternative approach for the coupler. Power entering the port 1 splits equally at the ports 2 through N and 2' through N'. It should be noted here that the characteristic impedance of the parallel and shunt lines shown in Fig. 2 (a) differs from that shown in Fig. 1 (a). The equivalent four-port network of Fig. 2 (a) is given in Fig. 2 (b).

Let us first examine the nonsymmetrical four-port directional coupler shown in Fig. 1 (b). It is seen that the four-port equivalent circuit has end to end symmetry and side by side asymmetry. To obtain a specified power splitting performance, as shown in Fig. 1 (b), it is necessary to derive explicit expressions for the impedances of both the parallel and shunt lines. By choosing two signals ( $E_a^+$ ,  $E_b^+$ ) = ( $\frac{1}{2}t$ ,  $\frac{1}{2}t$ ) applied at port 1 and two different signals ( $E_a^-$ ,  $E_b^-$ ) = ( $1/2t$ ,  $-1/2t$ ) applied at port 2 in Fig. 1 (b), where t is an arbitrary real constant to be determined later, the circuit can then be dissected by a properly chosen magnetic or electric wall on the basis of the even and odd mode method<sup>1</sup>.

A magnetic wall placed between the parallel lines is generally defined by a voltage maximum occurring at a point on the shunt branch lines with the even mode excitation of  $E_a^+$  and  $E_b^+$ . The even mode case is shown in Fig. 3 (a). The magnetic wall is located along the shunt lines at the point where:

$$\theta = \text{arc tan } \frac{E_b^+}{E_a^+} = \text{arc tan } \frac{1}{t} \quad (1)$$

Likewise, the position of electric wall is defined by a voltage minimum occurring at a point on the shunt branch lines with the odd mode excitation of  $E_a^-$  and  $E_b^-$ . The odd mode case is shown in Fig. 3 (b). The corresponding electric wall position is given by

$$\phi = \text{arc cot} \left( - \frac{E_b^-}{E_a^-} \right) = \text{arc cot} \frac{1}{t} \quad (2)$$

Now the ABCD matrix for the even mode (++) and odd mode (+-) case can be written in the following forms:

$$M_{++}^u = \begin{bmatrix} A_{++}^u & B_{++}^u \\ C_{++}^u & D_{++}^u \end{bmatrix} = \begin{bmatrix} \frac{(N-1)Z_1}{\frac{1}{Z_3}t} & jZ_1 \\ j\left[\frac{1}{Z_1} - \frac{Z_1(N-1)^2}{Z_3^2 t^2}\right] & \frac{(N-1)Z_1}{\frac{1}{Z_3}t} \end{bmatrix} \quad (3)$$

$$M_{+-}^1 = \begin{bmatrix} A_{+-}^1 & B_{+-}^1 \\ C_{+-}^1 & D_{+-}^1 \end{bmatrix} = \begin{bmatrix} \frac{Z_2 t}{Z_3} & jZ_2 \\ j\left[\frac{N-1}{Z_2} - \frac{Z_2 t^2 (N-1)}{Z_3^2}\right] & \frac{Z_2 t}{Z_3} \end{bmatrix} \quad (4)$$

where the superscripts u and 1 represents the upper and lower transmission paths in Fig. 3 (a) and (b).

The conditions for infinite directivity and perfect impedance matching in both the even and odd mode are

$$B_{++}^u = C_{++}^u \quad (5a)$$

$$B_{+-}^1 = \left( \frac{1}{N-1} \right)^2 C_{+-}^1 \quad (5b)$$

Substituting the corresponding terms given in Eqs. (3) and (4) into Eqs. (5a) and (5b), we obtain

$$Z_1 = \frac{Z_3 t}{\sqrt{(N-1)^2 + Z_3^2 t^2}} \quad (6a)$$

$$Z_2 = \frac{Z_3}{\sqrt{Z_3^2 + t^2}} \quad (6b)$$

The power coupling ratios at ports 3 and 4, as shown in Figure 1(b), are derived from the conditions for match and perfect directivity as

$$\frac{P_3}{P_1} = \frac{1}{N} = \frac{Z_3^4 t^4}{(N-1)^4 Z_1^2 \left[ 1 + \left( \frac{Z_3 t}{N-1} \right)^2 \right]^2} \quad (7)$$

$$\frac{P_4}{P_1} = \frac{N-1}{N} = \frac{Z_3^2 (N-1)}{Z_2^2 (Z_3^2 + t^2)^2} \quad (8)$$

Furthermore, from Eqs. (6), (7) and (8), the explicit expressions for the characteristic impedances can be determined and they are

$$Z_1 = Z_2 = \frac{1}{\sqrt{N}} , \quad Z_3 = 1, \quad t = \sqrt{N-1} \quad (9)$$

So far we have derived the design formulas for the first type of the N-way symmetrical branch line coupler. The second type of the coupler shown in Figure 2(a) can also be analyzed on the basis of the previous analysis. The only difference is that the equivalent four-port network shown in Figure 2(b) has end to end asymmetry and side-by-side symmetry. We omit the details and the explicit expressions for the impedances of the second type coupler are as follows:

$$Z_1' = \sqrt{\frac{N-1}{2}} , \quad Z_2' = 1 \quad (10)$$

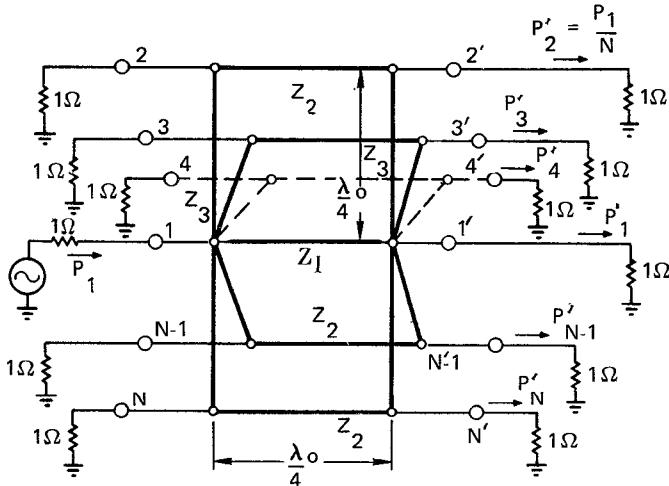
Note that the results given in Eqs. (9) and (10) for the case of N=3 agree with those given by Kuroda, et al.<sup>4</sup>.

To substantiate the analytical results for the N-way branch line coupler, a 3-way coupler having impedance level of 50 ohms was constructed and tested. The nominal center frequency was 2.42 GHz. It was constructed in microstrip using 1/16-inch thick teflon fiberglass board. In Figure 4, the measured results of the coupler are shown. The VSWR at input port is seen to be quite low. It is less than 1.25 over the frequency range from 2.36 to 2.48 GHz. The measured isolation between ports 4 and 6 at 2.42 GHz was approximately 12 dB. From the measured results, it can be seen that the useful bandwidth of the coupler is less than five percent.

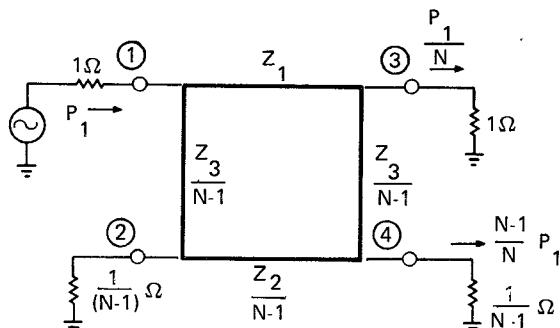
In conclusion, design formulas for N-way branch line couplers have been derived based on the equivalent even and odd mode networks of the four-ports. It is believed that broadband N-way multi-section branch line couplers can also be worked out by combining the analysis described herein with the synthesis procedure given by Levy and Lind<sup>3</sup>.

#### References

1. J. Reed and G. J. Wheeler, "A Method of Analysis of Symmetrical Four-Port Networks", IRE Trans. on MTT, Oct. 1956, pp. 246-252.
2. L. Young, "Branch Guide Directional Coupler", Proc. Natl. Electronics Conf., Chicago, Illinois, Oct. 1-3, 1956; Natl. Electronics Conf., Chicago, Ill., Vol. 12, April 15, 1957, pp. 723-732.
3. R. Levy and R. Lind, "Synthesis of Symmetrical Branch - Guide Directional Couplers", IEEE Trans. on MTT, Vol. 16, No. 2, Feb. 1968, pp. 80-89.
4. T. Kuroda, T. Usui, and R. Yano, "Multi-Port Lattice - type Hybrid Network", Presented at the International Microwave Symposium, Washington D.C., May 17, 1971.

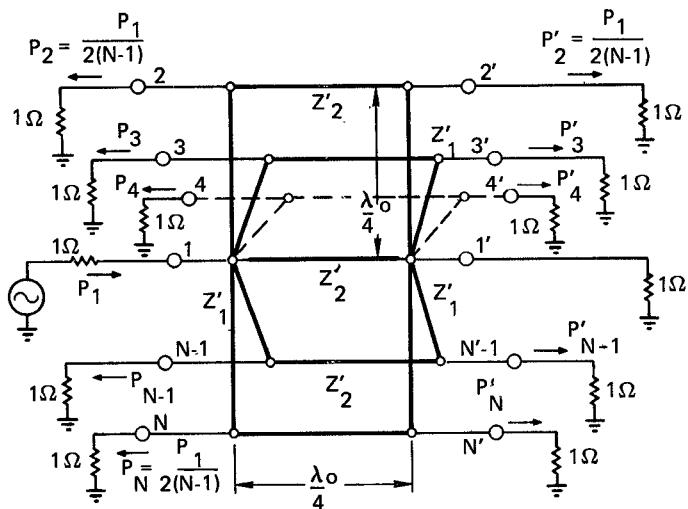


(a) First type of N-way directional coupler (N output ports).

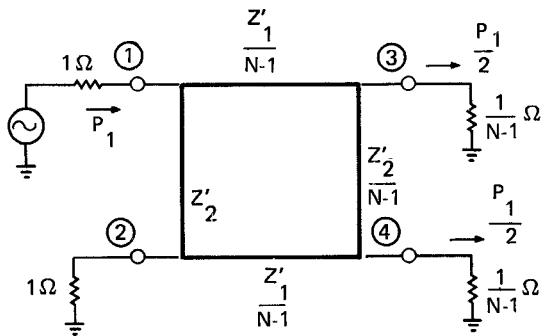


(b) Equivalent Four-Port Network.

Figure 1.

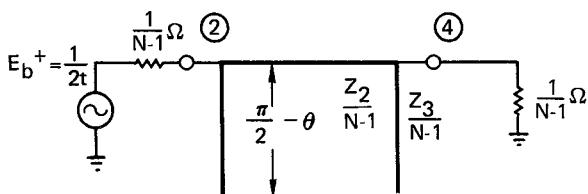
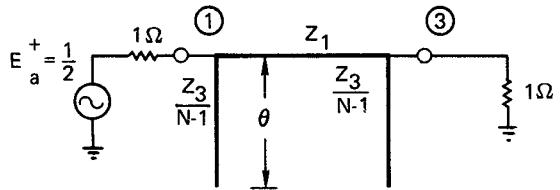


(a) Second type of N-way directional coupler [2 (N-1) Output Ports].

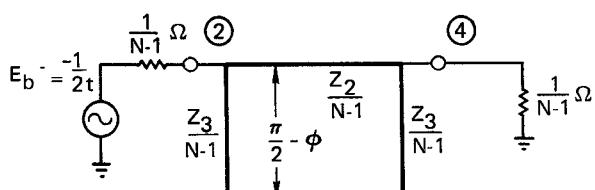
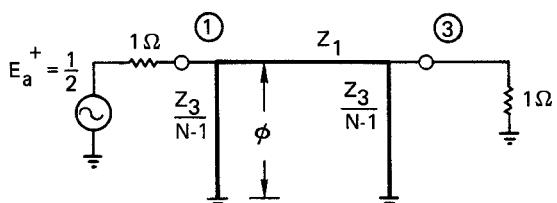


(b) Equivalent Four-Port Network.

Figure 2.



(a) Even Mode Networks.



(b) Odd Mode Networks.

Figure 3.

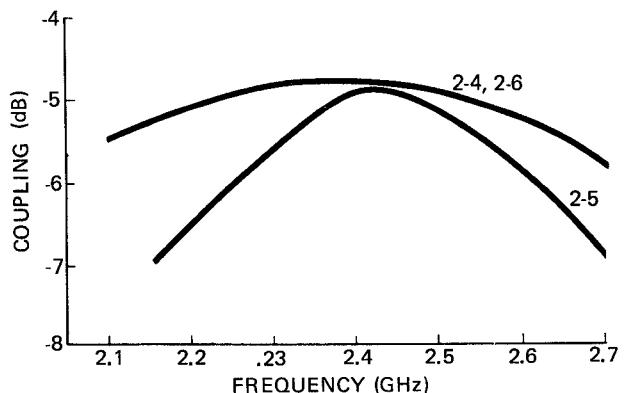
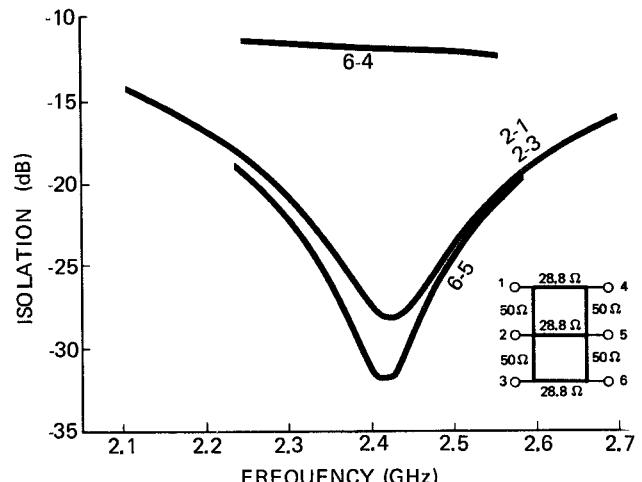
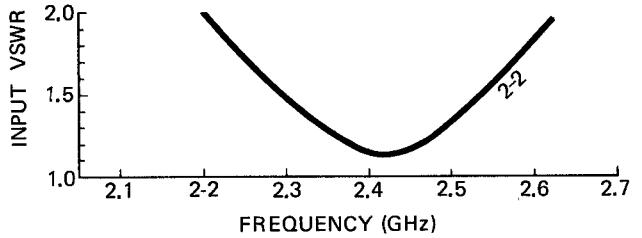


Figure 4. Experimental Results for a 3-Way Coupler

N O T E S